Observation Impact

Langland and Baker 2004 developed adjoint method to evaluate the impact of an arbitrary subset of observations on some forecast metric, e.g. RMSE.

In Kalman filter, the sensitivity of analysis x^a to observation y^o can be expressed as

$$\frac{\partial x^a}{\partial y^o} = K \qquad \text{since } x^a = x^b + K(y^o - h(x^b))$$

This means the Kalman gain maps changes in observation space to state space. The sensitivity of response function $J(x_t)$ to observations y^o can be expressed as

$$S_y^T = \frac{\partial J}{\partial y^o} = \underbrace{\frac{\partial J}{\partial x^a}}_{S_o^T} \underbrace{\frac{\partial x^a}{\partial y^o}}_{K} \qquad x^a \text{ is valid at } t=0$$

$$\delta J = \frac{\partial J}{\partial x_t} \frac{\partial x_t}{\partial x_o^a} \frac{\partial x_o^a}{\partial y^o} \delta y^o = S_y^T \delta y^o$$

$$\rightarrow \quad S_y = K^T \tilde{M}_t^T S_t$$

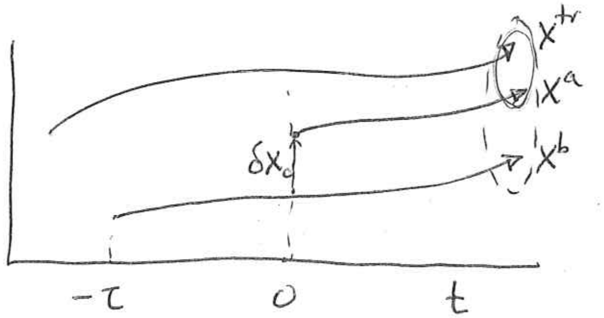
K^T maps sensitivity gradients from state space back to observation space.

Define $J(x_t)$ as an error statistics in model domain at time t.

$$J(x_t) = \frac{1}{2}(x_t - x_t^{tr})^T C (x_t - x_t^{tr})$$

where C normalizes the forecast error, x^{tr} comes from truth or reference state.

Now, picture the model trajectories from forecasts initialized from $t=-\tau$ and $t=0$, where data assimilation occurred at $t=0$ using the forecast from $t=-\tau$ as background.



To estimate how y^o at time $t=0$ produce changes in $J(x_t)$, errors at some later time during forecast

$$\Rightarrow \delta J = J(x_t^a) - J(x_t^b)$$

$$\cong \frac{\partial J}{\partial x_0} \underbrace{\delta x_0}_{(x^a - x^b)}$$

$\delta J > 0$ error increases.
$\delta J < 0$ " reduces

$$2\delta J = (x_t^a - x_t^{tr})^T C (x_t^a - x_t^{tr}) - (x_t^b - x_t^{tr})^T C (x_t^b - x_t^{tr})$$

$$= (x_t^a - x_t^b)^T \Big[\underbrace{C(x_t^a - x_t^{tr})}_{\frac{\partial J(x_t^a)}{\partial x_t^a}} + \underbrace{C(x_t^b - x_t^{tr})}_{\frac{\partial J(x_t^b)}{\partial x_t^b}} \Big]$$

Note that $\dfrac{\partial J(x_t)}{\partial x_t} = C(x_t - x_t^{tr})$

Given x_t^a, x_t^b, x_t^{tr}, $2\delta J$ can be exactly calculated.

To further express δJ as a function of observations' changes $\delta y^o = y^o - h(x^b)$:

$$2\delta J = (\tilde{M}_t \delta x_0)^T \left(\frac{\partial J(x_t^a)}{\partial x_t^a} + \frac{\partial J(x_t^b)}{\partial x_t^b} \right)$$

$$= \delta x_0^T \left(\tilde{M}_t^T \frac{\partial J(x_t^a)}{\partial x_t^a} + \tilde{M}_t^T \frac{\partial J(x_t^b)}{\partial x_t^b} \right)$$

$$= \left[K(y^o - h(x^b)) \right]^T \left(\frac{\partial J(x_t^a)}{\partial x_0^a} + \frac{\partial J(x_t^b)}{\partial x_0^b} \right)$$

$$= \delta y^o K^T S_0^{a,b} = \delta y^o K^T \tilde{M}_t^T (S^a + S^b)$$

how y^o contribute to reducing error

A sensitivity vector S^a, S^b can be mapped back to $\dfrac{\partial J}{\partial y^o}$ using K^T and $\tilde{M}_t^T$

Ensemble Forecast Sensitivity to Observations

(EFSO) (Liu and Kalnay 2008)

Again, using ensemble we can avoid the use of adjoint in estimating FSO: $\frac{\partial J}{\partial y^o}$.

Similar to ensemble sensitivity, estimate $\frac{\partial J}{\partial x}$ with

$\mathbb{E}(\delta J\, \delta x^T)\, \mathbb{E}(\delta x\, \delta x^T)^{-1}$ $\Rightarrow$ within sampling errors, this will give same results as adjoint method.

$$K^T \tilde{M}_t^T = (\tilde{M}_t P^a H^T R^{-1})^T$$

Note: $K = P^b H^T (H P^b H^T + R)^{-1} = P^a H^T R^{-1}$

$$\approx \left[\tilde{M}_t \mathbb{X}^a \mathbb{X}^{aT} H^T R^{-1} \right]^T$$

where $\mathbb{X}^a = \frac{1}{\sqrt{N-1}} \left(x_1^{a\prime}\ x_2^{a\prime} \cdots x_N^{a\prime} \right)$

$$\tilde{M}_t \mathbb{X}^a = \mathbb{X}_t^b = \frac{1}{\sqrt{N-1}} \left(x_{t,1}^{b\prime}\ x_{t,2}^{b\prime} \cdots x_{t,N}^{b\prime} \right), \quad x_{t,k}^{b\prime} = m(x_k^a) - \overline{m(x^a)}$$

$$(H\mathbb{X}^a)^T = \mathbb{Y}^{aT} = \frac{1}{\sqrt{N-1}} \left(y_1^{a\prime}\ y_2^{a\prime} \cdots y_N^{a\prime} \right)^T, \quad y_k^{a\prime} = h(x_k^a) - \overline{h(x^a)}$$

$$K^T \tilde{M}_t^T = \left(\underbrace{\mathbb{X}_t^b\, \mathbb{Y}^{aT}} R^{-1} \right)^T$$

covariance between analysis ensemble in observation space and forecast ensemble state.

$\Rightarrow$ FSO is more efficient than performing a full Observing System Simulation Experiment (OSSE) to evaluate observation impact.

Targetted observation (Xie et al. 2013)

Now we have methods to establish relation between current observations to future forecast model states. We can use this connection to perform adaptive sampling of the current model state $\Rightarrow$ Targetted observation.

Idea: calculate sensitivity of forecast to observation (FSO) and observe the location / region with high sensitivity.

Consider assimilation of potential observations will reduce error covariance at time $t=0$:

$$P^b - P^a = KHP^b = P^b H^T (HP^b H^T + R)^{-1} HP^b$$

at forecast time t the covariances become:

$$\begin{aligned} P^b_t - P^a_t &\approx \tilde{M}_t (P^b - P^a) \tilde{M}_t^T \\ &= \tilde{M}_t X^b X^{bT} H^T (H X^b X^{bT} H^T + R)^{-1} H X^b X^{bT} \tilde{M}_t^T \\ &= X^b_t Y^{bT} (Y^b Y^{bT} + R)^{-1} Y^b X^{bT}_t \end{aligned}$$

evaluate changes in X^b_t due to assimilation of observation y^o

$$\delta x^b_t = X^b_t Y^{bT} (Y^b Y^{bT} + R)^{-1} \left[y^o - h(x^b_0) \right]$$

$\Rightarrow$ Select y^o so that δx^b_t is desired in terms of reducing forecast error!