Hybrid Methods

Summary of Data Assimilation Methods

In general form: $W = P^b H^T (H P^b H^T + R)^{-1}$ (1)

$\delta x = W d^{o-b}$ (2)

$P^a = (I - WH) P^b$ (3)

an alternative of (2): $\left[(P^b)^{-1} + H^T R^{-1} H \right] \delta x = H^T R^{-1} d^{o-b}$ (4)

Method	How P^b is modeled	Solver	Pros	Cons
OI	B from climatology	directly solve (2)	full-rank B	B is static isotropic
3DVar	"	cost fcn minimizing (4)	"	"
Extended KF	B propagated by M, $M^T \to P^b$	directly solve (2), (3)	full-rank flow-dependent	cost too much not feasible!
EnKF	P^b estimated from ensemble	directly solve (2), (3)	flow-dependent	rank deficient (N)
4DVar	B propagated by M, $M^T \to P^b$	cost fcn minimizing $(4)^+$	full-rank with some flow dependency	costly to maintain M, M^T

Pure ensemble-based methods (EnKF):

- ensemble size N is not large enough to represent the true forecast errors efficiently (rank-deficient)
- Model errors and hidden error sources in observing networks are not necessarily accounted for.

$\Rightarrow$ a hybrid data assimilation method combines the full-rank climatological B and the flow-dependent ensemble-estimated P^b in variational framework. (4DVar)

$$P^{hyb} = (1-\beta) B + \beta P^b \qquad \beta \in [0,1] \qquad (5)$$

Introduce P^b into variational framework through "α-control" variables during preconditioning. (Lorenc 2003)

$\breve{u} = \begin{pmatrix} \vec{v} \\ \vec{\alpha} \end{pmatrix}$ is the extended control variable

$\tilde{L} = \left(\sqrt{1-\beta}\, L \quad \sqrt{\beta}\, X^b \right)$ is the extended preconditioner

$$\delta x_0 = \tilde{L} u = \sqrt{1-\beta}\, L v + \sqrt{\beta}\, X^b \alpha \qquad (6)$$

cost function becomes

$$J(u) = \frac{1}{2} u^T u + \frac{1}{2} \sum_{\tau=0}^{t} \left(d_\tau^{o-b} - H_\tau \tilde{M}_\tau \tilde{L} u \right)^T R_\tau^{-1} \left(d_\tau^{o-b} - H_\tau \tilde{M}_\tau \tilde{L} u \right) \qquad (7)$$

$$\nabla_v J = 0 = v + \sqrt{1-\beta} \sum_{\tau=0}^{t} (H_\tau \tilde{M}_\tau L)^T R_\tau^{-1} (H_\tau \tilde{M}_\tau \delta x_0 - d_\tau^{o-b}) \qquad (8)$$

$$\nabla_\alpha J = 0 = \alpha + \sqrt{\beta} \sum_{\tau=0}^{t} (H_\tau \tilde{M}_\tau X^b)^T R_\tau^{-1} (H_\tau \tilde{M}_\tau \delta x_0 - d_\tau^{o-b}) \qquad (9)$$

$(8) \cdot \sqrt{1-\beta}\, L + (9) \sqrt{\beta}\, X^b$:

$$0 = \underbrace{\sqrt{1-\beta}\, L v + \sqrt{\beta}\, X^b \alpha}_{\delta x_0} + \underbrace{\left[(1-\beta) L L^T + \beta X^b X^{bT} \right]}_{P^{hyb}} \sum_{\tau=0}^{t} \tilde{M}_\tau^T H_\tau^T R^{-1} (H_\tau \tilde{M}_\tau \delta x_0 - d_\tau^{o-b}) \qquad (10)$$

E4DVar: 2-way coupling between EnKF and 4DVar

1. run ensemble forecast to get x_k^b, $k=1,2,\cdots,N$
2. Calculate $\bar{x}^b$ and $x_k^{b'}$, $k=1,2,\cdots,N \Rightarrow \mathbb{X}^b$
3. use EnKF to update $x_k^{a'} = x_k^{b'} - KHx_k^{b'}$
4. run 4DVar with P^{hyb} as in (10) using $\bar{x}^b$ as prior $\Rightarrow$ analysis $\bar{x}^a = \bar{x}^b + \delta x_0$
5. recenter posterior ensemble $x_k^a = \bar{x}^a + x_k^{a'}$, $k=1,2,\cdots,N$
6. step forward in time and cycle through 1-5

4DEnVar: replace functionality of $\tilde{M}$ and $\tilde{M}^T$ with
(Liu et al. 2008) 4D ensemble trajectories $\Rightarrow$ no need to maintain TLM, ADM codes.

similar work flow to E4DVar, but in (10)

$$H_\tau \tilde{M}_\tau \mathbb{X}^b = H_\tau \tilde{M}_\tau \left(x_1^{b'}\; x_2^{b'} \cdots x_N^{b'} \right) / \sqrt{N-1}$$

$$\approx \frac{1}{\sqrt{N-1}} \left\{ \left(h_\tau[m_\tau(x_1^b)]\; h_\tau[m_\tau(x_2^b)] \cdots h_\tau[m_\tau(x_N^b)] \right) - \overline{h_\tau(m_\tau(x^b))} \right\}$$

Consider how to apply localization ρ_L to ensemble P^b in (10):

$$\text{E4DVar}: \left[(1-\beta)B + \beta(\rho_L \circ P^b) \right] \sum_{\tau=0}^{t} \tilde{M}_\tau^T H_\tau^T R_\tau^{-1} \left(H_\tau \tilde{M}_\tau \delta x_0 - d_\tau^{o-b} \right)$$

localize P^b at time 0, then propagate in time the localized P^{hyb}

$$\text{4DEnVar}: \cdots + \sum_{\tau=0}^{t} \beta\, \rho_L' \circ \left[\mathbb{X}^b (H_\tau \tilde{M}_\tau \mathbb{X}^b)^T \right] R^{-1} \left(H_\tau \tilde{M}_\tau \mathbb{X}^b \alpha + H_\tau L v - d_\tau^{o-b} \right)$$

need to figure out how to localize a temporal covariance.

$\rightarrow (1-\beta)B$ part is not propagated in time either.

Note on localizing P^b:

Buehner 2005: $\mathbf{X}_L^b = \frac{1}{\sqrt{N-1}} \left(\mathrm{diag}(x_1^{b'}) \rho_L^{\frac{1}{2}},\ \mathrm{diag}(x_2^{b'}) \rho_L^{\frac{1}{2}},\ \cdots,\ \mathrm{diag}(x_N^{b'}) \rho_L^{\frac{1}{2}} \right)$

so that $\mathbf{X}_L^b \mathbf{X}_L^{bT} = \rho_L \circ P^b$

Compare 4DEnVar and E4DVar:

Analysis increment at time t

$$\delta x_t = \tilde{M}_t \delta x_0$$

$$= \sqrt{1-\beta}\, \tilde{M}_t L v + \sqrt{\beta}\, \tilde{M}_t \mathbf{X}^b \alpha$$

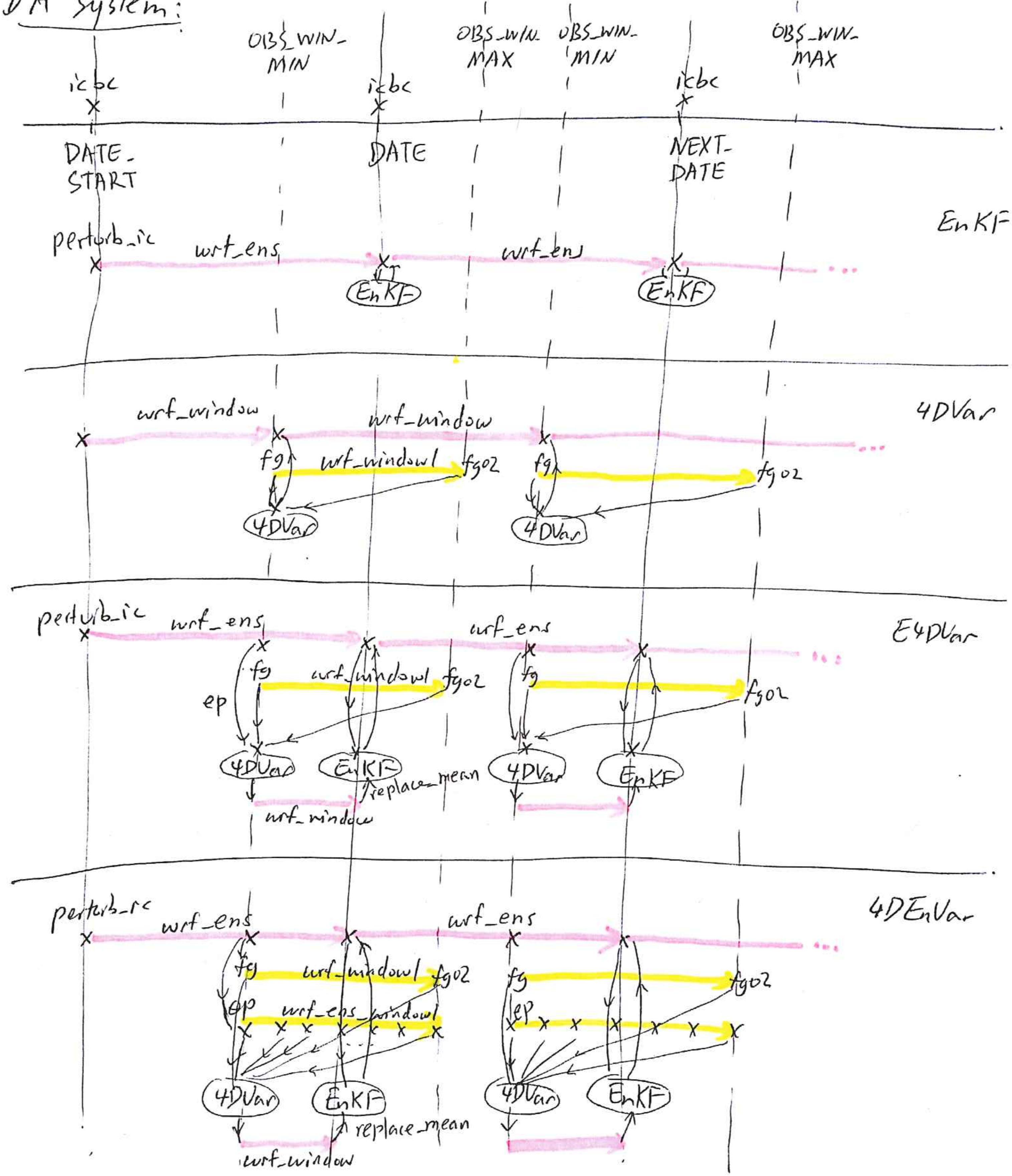
for E4DVar, if $\beta = 0$, return to 4DVar.

for 4DEnVar, no $\tilde{M}_t$ is available.

$$\delta x_t = \sqrt{1-\beta}\, L v + \sqrt{\beta}\, \mathbf{X}_t^b \alpha$$

if $\beta = 0$, becomes 3D-FGAT.

DA system:



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